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Welland, Grant V.; University of Missouri-St. Louis
DeFacio, Brian; University of Missouri-Columbia

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Dr. Arje Nachman
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13. ABSTRACT (*Maximum 200 words*)

This is the final report on the project entitled **Wavelets and Scattering** supported under AFOSR grant 90-307. During this project wavelets were used to analyze several problems in signal processing, quantum optics, elastic wave nondestructive evaluation, electromagnetic scattering and the dielectric response of water. The grant supported two students. One of them, Dr. C. R. Thompson is now an AF Captain in the ODEL Division at Brooks AFB in Texas.

A number of research papers were published including the first calculation of p-wavelets. Another publication shows the scale change of wavelet theory corresponds to the squeezing operation in quantum optics. A wavelet approach to visual recognition of faces was completed and has been submitted for publication. The Calderón-Grossmann-Morlet reproducing formula was shown to hold for the two-sided ideal of Hilbert-Schmidt operators. In elastic wave NDE, the frequency scales in phase space for the front face echo were shown to require a very different compression from the other scales. New results on Maxwell's equations in regions with Lipschitz boundaries were published.

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Brian DeFacio
Department of Physics and Astronomy
University of Missouri
Columbia, MO

Grant V. Welland
Department of Mathematics and Computer Science
University of Missouri--St. Louis
St. Louis, MO

June 8, 1994

INTRODUCTION

This is the final project report for AFOSR grant 90-0307, Wavelets and Scattering. The original goals of this proposal included:

- Obtaining a better understanding of wavelets, including their strengths and weaknesses in areas which lie in the mission of AFOSR.
- Applying wavelets to electromagnetic scattering problems, ultrasonic nondestructive evaluation, quantum optics, signal analysis and the dispersive structure of the dielectric response of water.

Wavelets are very promising mathematical and computational objects. A single function, together with its dilations (translations and scale changes), generates a basis for function spaces used in many common physical situations. Scale changes provide a multiresolution structure which clearly and concisely describes elements of a solution or operator at different frequencies and thereby unifies diverse areas and applications. This facilitates the transfer of methods and techniques among problems, which explains the appeal of wavelet methods. Some of their useful properties include data compression, noise reduction, edge detection, and feature extraction.

In the next section, Publications, the 1994 publications which appeared, are in press, or submitted are listed and comments are given on what has been learned. In the section on Follow - On Projects, the next studies which are made possible are presented. The last section gives the Conclusion.

PUBLICATIONS 1994

Welland:

W1 G. V. Welland and M. Lundberg Construction of Compact p-Wavelets, Constructive Approximation, 9, 347-370(1993).

W2 R. Torres and G. V. Welland, The Helmholtz Equation and Transmission Problems with Lipschitz Interfaces, Indiana Univ. Math. J. 42, 4, 1457-1485.

W3 M. Mitrea. R. Torres and G. V. Welland, Regularity and approximation results for the Maxwell problem on C^1 and Lipschitz domains,. To appear in the proc. of conf. on Clifford Algebras held in April 1993, (12 typed pp).

W4 M. Mitrea. Torres and G. V. Welland, Layer potential techniques in electromagnetism, submitted to J. *of Int. Eq. and Applic.* (29 pp)

W5 S. K. Bhatia, V. Lakshminarayanan, A. Samal, G. V. Welland Parameters for Human Face Recognition submitted to J. of Visual Communicaton and Image Representation

(26 typed pp)

DeFacio:

D1 C. R. Thompson and B. DeFacio, Two-dimensional image analysis using the wavelet transform, in Inverse Problems in Scattering and Imaging, SPIE 1767, Editor, M. Fiddy, (SPIE, Bellinghan WA) (1992) 120 - 130.

D2 B. DeFacio and S.-H. Kim, Non-uniqueness in direct and inverse electromagnetic scattering theory, in Inverse Problems in Scattering and Imaging SPIE 1767, Editor, M. Fiddy, (SPIE, Bellingham WA, 1992), pp 21-30.

D3 C.R. Th..pson and B. DeFacio, Information-to-noise improvement in the frequency domain using the wavelet transform, in Inverse Problems in Scattering and Imaging, 1767, Editor, M. Fiddy (SPIE, Bellingham WA, 1992) pp 131 - 146.

D4 D.M. Patterson, B. DeFacio, C.R. Thompson and S.P. Neal, Wavelets and their applications to digital signal processing in ultrasonic NDE, in Rev. Prog. QNDE, Edited by D.O. Thompson and D.E. Chimenti (Plenum, New York, 1993), pp 719 - 726.

D5 B. DeFacio, A. Van Nevel, and O. Brander, Double simple-harmonic-oscillator formulation of the thermal equilibrium of a fluid interacting with a coherent source of phonons, International Workshop on Harmonic Oscillators NASA Conf. Proc. 1621, Edited by D. Han, Y.-S. Kim and W. Zachary (NASA, Greenbelt MD, 1993) pp 309 - 322.

D6 S.-H. Kim, G. Vignale, and B. DeFacio, Frequency and wavevector dependent dielectric function of water-like fluids, Phys. Rev. A 46, 7548-7560 (1992).

D7 G.M. D'Ariano and B. DeFacio, A quantum wavelet for quantum optics, Il Nuovo Cimento B 108, 753-763 (1993).

D8 S.-H. Kim, B. DeFacio and G. Vignale, Refractive index of water-like fluids, Phys. Rev. E. 48, 3172-3175 (1993).

D9 B. DeFacio, Coherent-state path-integrals and their relation to wavelets, in a Festschrift for J. R. Klauder, Edited by G.G. Emch, G. Hegerfeldt and L. Streit (Springer-Verlag, New York, in press) 18 pp.

D10 S.-H. Kim, B. DeFacio and G. Vignale, The dynamic dieleltric response of liquid water, submitted to Phys. Rev. E.

D11 B. DeFacio, S.-H. Kim and A. VanNevel, Application of Squeezed States=Bogoliubov transformations to the statistical mechanics of water and its bubbles, International Workshop on Squeesed States and Uncertainty Relations, in NASA *Conf. Proc.* XXXX, edited by D. Han, and M. Rubin, Y. Shih and M.A. Man'kov (NASA, Greenbelt, in press) 13 pp.

D12 **D.M. Patterson and B. DeFacio, Wavelet Inversion of Data for Elastic Wave Nondestructive Evaluation, in Inverse Optics III,** *SPIE* **2241,** (SPIE, Bellingham, WA, in press), 13 pp.

D13 H. Kaiser, K. Hamcher, R. Kulasekere, W.-T. Lee, J.F.Anker, B. DeFacio, P. Miceli and D. L. Worchester, **Neutron Optics in Layered Materials in Inverse Optics III,** *SPIE* 2241 (*SPIE Bellingham WA*, in press), 12 pp.

FOLLOW-ON PROJECTS

- Continuation of work with (McDonnell-Douglas Corp. CEM group); Wavelet methods, matrix sparsening methods for 3-D elctromagnetic scattering. (GVW)
- Statistical mechanics of the dialectric response of water (BDF with G. Vignale, Cai and Welland).
- NDE using ultrasound, especially in composites, layered materials, bond adhesion (with Martin-Marietta, Baltimore).
- p-biorthogonal wavelet project with B. Jawerth with the boundary adjustments for edge sets. (GVW)
- feature extraction using wavelet packets for the development of a face recognition system with the target system to be competitive with the human capability for face recognition.
- Novel dielectrics including microwave Penrose tiling structures, wire structures, periodic arrays of spheres, honeycomb structures. The band-gap or pseudo-gap, switching properties, defect and the dispersion will be studied for each of these new materials (BDF with Satpathy).
- Inverse Neutron Optics of Layered Materials (BDF with H. Kaiser and D. Worchester).

CONCLUSION

The study of the frequency dependence of the dielectric response of water is a first step towards a quantitative understanding of the propagation of electromagnetic waves in tissue. Such understanding is required to answer

health and safety questions what are the responsibility of the Radiation Analysis Division of the Laboratory at Brooks AFB in Texas. The squeezed states developed for quantum optics will be studied for noise reduction in NDE. It is expected that this work will enhance the collaboration of BDF with scientists at Martin-Marietta. The study of bubble formation and their decay addresses questions concerning the health and safety of ultrasound in medicine, since these decays are either via shock waves, or initially high velocity jets of gas and matter.

Numerical work based on multileveling, sparsening, and conditioning methods are used in electromagnetic scattering problems which are very difficult to resolve because of technical problems. The technical problems derive from large high density matrices which require inversion. Large matrices occur in cases for which there is a large ratio of the characteristic length of the scattering body to wavelength of an illuminating source in problems involving body element methods. This is at the heart of the collaboration of GVW with Lou Mitschang, D.-S. Wang, et al. at McDonnell-Douglas (MDA). This has been extended to the use of local SVD methods and the use of wavelet packets to provide a controlled sparsening. The goal is to be able to solve very large problems using sparsening techniques in conjunction with other available methods or methods under development, while maintaining an understanding of implications with respect to invertibility and reliability of solutions obtained using sparsening techniques. These methods are to be developed with the objective for more general applicability. Other groups at MDA are involved in this project, e.g. the CFD group.

Methods using wavelet packets are being developed for application to a machine based recognition system for human faces. This is the first step in a project to obtain good representation of this class of images. One must clearly distinguish recognition from identification. The goal of this effort will be to develop a system that rivals the power of the human system reported in the paper, **Parameters for Human Face Recognition.** The methods will require registration of images through shifts and rotations, which requires a generalization of existing methods. This problem gathers together all of the ideas in the present project, and more.

These techniques will also play an important role in simulation of statistical mechanics models and, in particular, in the simulation of the dielectric response of water to electromagnetic pulses. Our studies are ongoing and we are making substantial progress toward the goals stated above.

The studies completed, so far, have developed methods and understanding which uniquely qualify the PI's for these follow-on studies. The follow-on

studies will incorporate refinement of analytic modeling of physical problems and numerical simulation.

The publications and preprints listed in section 2 add to the understanding of several problems of interest to the AFOSR. The p-wavelets in (WI) provides new possibilities for segmenting, compression and denoising signals and one of us (GVW) is using these wavelets to study edge sets with Prof. B. Jawerth. The papers on the dispersion of water, which appear in references, [D6], [D8], [D10] and [D11] show established structures such as the Debye decrease in dielectric response function, and such potentially new effects which have not yet been experimentally observed such as the collective dipolaron mode. A free rotor peak was found in the far infrared frequency region at exactly the frequency of the Simpson et al. peak which was experimentally reported, but had not been obtained in other theoretical models. However, the peak which was found was much too large and quantitative agreement has not been obtained. The dipolaron, if correct, would totally change the accepted nature of the relaxation mechanism. This collected mode cannot be ruled out by the data which exists today.

The publication listed under [D7] presented one operator-valued wavelet in quantum optics. There are two kinds of low-noise quantum states in optics, coherent states and squeezed states, and it is well known no additional such objects are possible. In a group theoretic approach, the coherent state is a translation in Fock space. In this paper, the scale change of wavelet theory is shown to be implemented by the squeeze operator. This opens several paths for study of noise in scattering, in addition to optics.

The publications listed in [D4], [D12] and [D13] apply wavelet analysis to inverse scattering of elastic waves for NDE and to glancing angle thermal neutrons from layered materials including both high technology metal composites and lipid bilayers.

We take the opportunity to point out that Brian DeFacio was elected to Fellow of the American Physical Society in November, 1993.

We have not applied nor obtained any patents. We have made no inventions. The support of this research by AFOSR is greatly appreciated by both of the PI's.

Brian DeFacio
Brian DeFacio
Professor

Grant Welland
Grant Welland
Professor

Inverse Problems in Scattering and Imaging
SPIE 1767 - 4, San Diego, CA, July 19-22, 1992

Non-uniqueness in direct and inverse electromagnetic scattering theory

B. De Facio and S.-H. Kim
Missouri University, Department of Physics
Columbia, MO 65211

Abstract

General statements of impossibility can be important in science and engineering. Ambiguities in inverse problems are cases of non-uniqueness where classes of different objects give the same response. A *strong ambiguity* is one which no additional data will remove the non-uniqueness, whereas a *weak ambiguity* is one which can be removed by additional data. In direct scattering theory, different potentials with one or more trapped modes may give the same $\mathrm{R}(k)$ or the $e^{i\alpha}\mathrm{R}(k)$ where α is a real parameter at all wave-numbers k. In three-dimensional direct scattering theory, different material media and sources $\vec{J}$, ρ give the same scattering matrix at all times (or wave numbers) at all scattering angles and all incident angles. Examples of strong ambiguities will be given including one where a temporal relaxation of a homogeneous body is equivalent to a totally different time-independent homogeneous body. Weak ambiguities will be presented including both examples of incident scatters. The conditions on the scatterers at spatial infinity and their trapped mode bound-state structure will be given.

1. INTRODUCTION

The objective of inverse problems[1-4] is to study families of models for solutions in a known class, which are amenable computation. Given a set of noise-free data the question is which, if any, stable solution exists in the class, is it unique and how it can be calculated to a useful accuracy (in a reasonable amount of time on an available computer). If the data set is "too small" because of sampling or band-limiting, or if the signal is corrupted by noise, the problem becomes ill-posed. An ill-posed problem has a solution which depends discontinously on the initial data, so that it is also unstable. Most of the interesting applied inverse problems are ill-posed. At the 1979 Delaware Symposium on Ill-posed Problems, P.C. Sabatier defined interesting inverse problems as those which are ill-posed.

The class of models studied in this paper will be restricted to those in which the electromagnetic scattering of objects is described by Maxwell's equations in second order form. The class of non-uniqueness studied here was first discovered by Sabatier[5-8] in geophysical acoustic scattering and generalized to electromagnetism by De Facio[9]. Corones and Winter[10] studied the related homogenization properties of a scalar electomagnetic slab (1-d) problem using splitting methods. These ambiguities clarify the proofs of uniqueness for electromagnetic waves which are found in refs. (11 - 13). Sabatier's work was focused on discontinuities at interfaces between media. Strong dispersion is a fundamental property of the dielectric response of biological solutions[14,15]. Inverse problems in these dispersive media were reviewed in ref. (15). The physical origins of dispersion includes:

(1) non-trivial time fluctuations produce frequency ω dependence, called temporal dispersion, upon Fourier transformation,

(2) non-trivial spatial variation in an object gives wave-number $\vec{k}$ dependence, called spatial dispersion, upon Fourier transformation.

It is the temporal dispersion which is already well known to be important in biomathematics and at present the spatial dispersion has not yet been established experimentally. It will.

The identification of two distinct sets of coefficients with the same scattering amplitude in the direct problem is an ambiguity for the inverse problem.

Three dimensional inverse problems are highly overdetermined and the exact data set which is to be used requires careful consideration. For this reason, following Sabatier[3], non-uniqueness will be classified as a strong ambiguity if no possible additional data will remove it and as a weak ambiguity if there is possible additional information which will remove it. For example, near field information of the scattered field would remove the ambiguity given in ref. (16) but not that of ref. (17). If it is possible to measure the near field response, ref. (16) is a weak ambiguity and ref. (17) is a strong ambiguity. If this is not possible, then the objects in both references become strong ambiguities.

It is also possible to use specific examples of ambiguities to check the correctness of electromagnetic scattering computer programs, as pointed out by Dr. T. Roberts[18].

The organization of the rest of this paper is to present the Theorectical Framework in Sec. 2, Examples of the homogenenization, scaling ambiguities are presented in Sec. 3 and the Conclusions are given in Sec. 4. The Acknowledgements and References follow as Sec. 5 and 6, respectively.

2. THEORETICAL FRAMEWORK

A classical, linear electromagnetic wave at frequencies below 10^{13} Hz satisfies Maxwell's equations in Minkowski form[10]:

$$\vec{\nabla} \cdot \vec{D} = \rho_f, \tag{1a}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \tag{1b}$$

$$\vec{\nabla} \cdot \vec{B} = 0, \tag{1c}$$

$$\vec{\nabla} \times \vec{H} = \vec{J}_c + \frac{\partial \vec{D}}{\partial t}. \tag{1d}$$

The linear, isotropic, stationary constitutive relations are:

$$\vec{D} = \epsilon \vec{E} = \epsilon_0 \vec{E} + \vec{P} \tag{2a}$$

$$\vec{B} = \mu \vec{H} = \mu_0 \vec{H} + \mu_0 \vec{M} \tag{2b}$$

and

$$\vec{J}_f = \sigma \vec{E} \tag{2c}$$

where $(\vec{P}, \vec{M})$ are the electric polarization and the magnetization in matter and (ϵ, μ, σ) are the permittivity, permeability and conductivity of the matter. The electromagnetic field pair $(\vec{D}, \vec{B})$ has continuous normal componenets, the electric field $\vec{E}$ has continuous tangential components and the magnetic intensity $\vec{H}$ has coninuous tangential components except for the idealized $(\sigma \to \infty)$ a perfect conductor where the discontinuity in $\vec{H}$ at the interface $[\vec{H}]$ has the non-zero value

$$[\vec{H}] = [J_f] = \vec{K}_f, \tag{3}$$

where $\vec{K}_f$ is the surface current density.

The model to be studied here[9] is an electromagnetic pulse $(\vec{E}_0, \vec{H}_0)$ emitted in direction $\hat{e} \in S^2$ in a linear, isotropic, homogeneous, stationary (LIHS) medium Ω toward an obstacle which is either a wavy collection of layers or a collection of compact regions $\Omega_2, \Omega_3, \cdots, \Omega_{N+1}$ where

$$\Omega_2 \subset \Omega_3 \subset \cdots \subset \Omega_{N+1} \subset R^3. \tag{4}$$

The constitutive parameters $(\epsilon_i, \mu_i, \sigma_i)$ and shape characterize points in each Ω_i. Only Ω_1 is LIHS although $\Omega_2 \cdots \Omega_{N+1}$ will be taken as isotropic and stationary for simplicity. The inhomogeneity can be spatial or temporal or both. If the electromagnetic fields and sources $(\rho_f, \vec{J}_f)$ are in certain uniformly continuous Hölder spaces given in Colton and Kress[12] and used in ref. (9), then the scattering amplitudes all exist. The scattered fields $(\vec{E}, \vec{H})$ are given in ref. (9) decomposed into pure volume integral terms were called <u>diffuse</u> reflectors; surface terms integrated over discontinuities in $[\epsilon], [\mu]$ or $[\sigma]$ were called <u>soft</u> reflectors; and surface terms over gradients of discontinuous material parameters were called <u>hard</u> reflectors. The terminology was taken from geophysics. One consequence of this decomposition is that a fixed frequency pulse cannot distinguish amoung these three effects, whereas the high frequency part of a wide-band (narrow time) pulse will measure only the hard reflectors.

It is well-known that variable-coefficients partial differential equations (pde's), without analytic coefficients, are poorly understood even in their existence, uniqueness and stability

properties. The simple addition of spatial or temporal dependence adds two classes of possible non-uniqueness, that of homogenization and that of Sabatier scaling. The Sabatier scaling may be discontinuously or dispersion dependent and both classes can give either strong or weak ambiguities.

The Sabatier scaling for the present models[9] involve two functions (ϵ_s, μ_s) which can depend on t, $\vec{x}$ or both. However, it was shown[9] that $\epsilon_s \mu_s = 1$ is required. The t-dependence is particularly important for water and biological models in the microwave frequency range and has been the subject of much less investigation than the $\vec{x}$-dependent case. For the electromagnetic fields ($\vec{D}$, $\vec{E}$, $\vec{H}$, $\vec{B}$) and their sources (ρ_f, $\vec{J}_f$), the Sabatier scaling gives a set of fields ($\vec{D}^\sim$, $\vec{E}^\sim$, $\vec{H}^\sim$, $\vec{B}^\sim$) and sources ($\rho_f^\sim$, $\vec{J}_f^\sim$) defined as

$$\vec{D}^\sim = \frac{1}{\epsilon_s}\vec{D} = \mu_s \vec{D}, \tag{5a}$$

$$\vec{E}^\sim = \frac{1}{\epsilon_s}\vec{E} = \mu_s \vec{E}, \tag{5b}$$

$$\vec{H}^\sim = \mu_s \vec{H}, \tag{5c}$$

$$\vec{B}^\sim = \mu_s \vec{B}, \tag{5d}$$

with $\epsilon_s \mu_s = 1$ (pointwise). These scaling functions correspond to the multiplicative inverses of those in ref. (9), ϵ_s (here) $= 1/\epsilon_s$ (ref. 9), etc. The constitutive functions (ϵ_i, μ_i, σ_i) and the scaling function μ_s can depend on t alone, $\vec{x}$ alone or t and $\vec{x}$. The scaling for the sources (ρ_f, $\vec{J}_f$) is given by

$$\rho_f^\sim = \mu_s \rho_f = \frac{\rho_f}{\epsilon_s}, \tag{6a}$$

and

$$\vec{J}_f^\sim = \mu_s \vec{J}_f. \tag{6b}$$

We will deviate from ref. (9) by using a single scaling function, which one of us (S-HK) found to exhibit the structure in a clearer manner. Note that for non-trivial scaling $\mu_s(.) \neq 1$ in the interior of Ω_i ($i \neq 1$) and $\mu_s = 1$ (pointwise) everywhere in Ω_1.

The homogenization occurs when (ϵ_i, μ_i, σ_i) are functions of t or $\vec{x}$ and post-homogenization ambiguities occur when the scale transformations of eqs. (5a - d) are applied to Maxwell's equations. There are two general properties of the Sabatier scaling of Maxwell's electromagnetism which will be presented next.

Lemma 1. The pairs ($\vec{J}_f$, $\vec{E}$) and ($\vec{J}_f^\sim$, $\vec{E}^\sim$) have the same conductivity in this model, for all scaling parameters μ_s.

Proof Using (5b) and (6b) in eq (2c)

$$\vec{J}_f = \sigma \vec{E} = \epsilon_s \vec{J}_f^\sim = \sigma \epsilon_s \vec{E}^\sim$$
$$\vec{J}_f^\sim = \sigma \vec{E}^\sim.$$

∎

Lemma 2 If the scaled fields ($\vec{E}^\sim$, $\vec{H}^\sim$) inside Ω_i ($i \neq 1$) could be measured, all scaling ambiguities would be weak ambiguities.

Proof The Poynting vector for real fields $(\vec{E}, \vec{H})$ and $(\vec{E}^{\sim}, \vec{H}^{\sim})$ are given by

$$\vec{S} = \vec{E} \times \vec{H},$$

and

$$\vec{S}^{\sim} = \vec{E}^{\sim} \times \vec{H}^{\sim}.$$

Therefore,

$$\begin{aligned}\vec{S}^{\sim} &= \mu_s^2(\vec{E} \times \vec{H}) = \mu_s^2 \vec{S} \\ &\neq \vec{S}\end{aligned}$$

for $\mu_s \neq \pm 1$. ∎

Remarks

(1) On page 865 of ref. (13), a correct observation is made that different conducting dielectrics can have the same pointwise Maxwell equation solutions. The authors seem unaware of ref. (9) which includes this result as a special case. An elementary example will be given in the next section. Lemma 1 shows why this can occur structurally because one conductivity can correspond to different pairs $(\vec{J}_f, \vec{E})$ and $(\vec{J}_f^{\sim}, \vec{E}^{\sim})$ which are solutions to Maxwell's equations.

(2) The Poynting vector $\vec{S}$ is the power per unit area flowing in the direction of $\vec{S}$ at a point $\vec{x}$. The condition $\vec{S} \neq \vec{S}^{\sim}$ shows that a different power flows inside Ω_i $(i \neq 1)$ even though the external fields are identical. However, this lemma is inapplicable to remore sensing, astronomy and many objects of interest, such as inside a nuclear reactor because observations inside the object are impossible.

In the next section, the ambiguities will be given.

3. AMBIGUITIES

First, the homogenization structure of Maxwell's equations will be given for two cases: (μ_2, ϵ_2) depend only on time and then (μ_2, ϵ_2) depend only on space. The conductivity σ is carried in $\vec{J}_f$. The space dependent case has been far more frequently studied by authors.

Assuming $(\epsilon_2, \mu_2, \sigma_2)$ depend only on time t and taking the curl of eq. (1b), it is straightforward to use eqs. (1c), (1d) to obtain:

$$\Delta\vec{E} - \mu_1\epsilon_1\ddot{\vec{E}} = \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) + \partial_t(\mu_2\vec{J}_f) + [(\mu_2\epsilon_2 - \mu_1\epsilon_1)\ddot{\vec{E}} + (\partial_t(\mu_2\dot{\epsilon}_2))\vec{E}] + (\dot{\mu}_2\epsilon_2 + 2\mu_2\dot{\epsilon}_2)\dot{\vec{E}} \qquad (7a)$$

after adding a background term $-\epsilon_1\mu_1\ddot{\vec{E}}$ to both sides. Notations of $\dot{\vec{E}}$, $\dot{\epsilon}_2$ are used for partial derivatives with respect to time of a single vector or scalar, $\partial_t(\mu_2\vec{J}_f)$ is used for products and Δ is the Laplacian operator. A similar calculation for $\vec{H}$ gives

$$\Delta\vec{H} - \mu_1\epsilon_1\ddot{\vec{H}} = \vec{\nabla}(\vec{\nabla}\cdot\vec{H}) - \vec{\nabla}\times\vec{J}_f + [(\mu_2\epsilon_2 - \mu_1\epsilon_1)\ddot{\vec{H}} - (\partial_t(\dot{\mu}_2\epsilon_2))\vec{H}] + (\mu_2\dot{\epsilon}_2 + 2\dot{\mu}_2\epsilon_2)\dot{\vec{H}}. \qquad (7b)$$

Next, assume that $(\epsilon_2, \mu_2, \sigma_2)$ depend only on space $\vec{x}$, it is easy to take the curl of eqs. (1b), (1d) and to rearrange into

$$\Delta\vec{E} - \mu_1\epsilon_1\ddot{\vec{E}} = \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) + \mu_2\dot{\vec{J}}_f + [(\mu_2\epsilon_2 - \mu_1\epsilon_1)\ddot{\vec{E}}] + (\vec{\nabla}\mu_2)\times\dot{\vec{H}} \tag{8a}$$

and

$$\Delta\vec{H} - \mu_1\epsilon_1\ddot{\vec{H}} = \vec{\nabla}(\vec{\nabla}\cdot\vec{H}) - \vec{\nabla}\times\vec{J}_f + (\mu_2\epsilon_2 - \mu_1\epsilon_1)\ddot{\vec{H}} - \vec{\nabla}\epsilon_2\times\dot{\vec{E}}. \tag{8b}$$

The coupling between $\vec{E}$ and $\vec{H}$ in eqs. (8a), (8b) cannot be removed for general (μ_2, ϵ_2). At fixed frequency, the terms

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{E}) = \frac{1}{\epsilon_0}\vec{\nabla}[\rho_T], \tag{9a}$$

$$\begin{aligned}\vec{\nabla}(\vec{\nabla}\cdot\vec{H}) &= -\vec{\nabla}\left[\frac{1}{\mu_2^2}(\vec{\nabla}\mu_2)\cdot\vec{B}\right]\\ &= -\vec{\nabla}\left[\frac{1}{\mu_2^2}\vec{\nabla}[\mu_2]\cdot\vec{B}\right],\end{aligned} \tag{9b}$$

where eq. (1c) was used in eq. (9b) and ρ_T is the total charge density (bound plus free) can mix with additional terms to provide ambiguities. It is clear from eqs. (7) and (8) that fixed incident (i.e. scattering) contains little information on the details of the obstacle Ω_2.

For a metal or a conducting dielectric the time dependent $(\epsilon_2, \mu_2, \sigma_2)$ case the right hand side is either equal to or contains a term

$$\begin{aligned}[RHS] &= \partial_t(\mu_2\vec{J}_f)\\ &= (\dot{\mu}_2 J_f + \mu_2\dot{J}_f)\hat{n}_J\end{aligned} \tag{10a}$$

<u>Example 1.</u> A simple calculation shows that the pairs $(\mu_{20}, \vec{J}_{f0})$ and $(\mu_{21}, \vec{J}_{f1})$ are ambiguous, and the first pair gives a homogenization of the second when the directions of $\vec{J}_{f0}$, $\vec{J}_{f1}$ coincide and

$$\begin{aligned}\mu_{20} &= 1\\ J_{f0} &= e^{-\beta|t|}\\ \mu_{21} &= e^{-3\beta|t|/2}\\ J_{f1} &= e^{\beta|t|/2}.\end{aligned} \tag{10b}$$

Calculating

$$\begin{aligned}\partial_t(\mu_{21}J_{f1}) &= \dot{\mu}_{21}J_{f1} + \mu_{21}\dot{J}_{f1}\\ &= [(\frac{-3\beta}{2} + \frac{\beta}{2})e^{-3\beta|t|/2}]e^{\beta|t|/2}\\ &= -\beta e^{-\beta|t|} = \partial_t(\mu_{20}J_{f0}).\end{aligned}$$

Therefore, the pairs $(\mu_2, J_f) = (1, e^{-\beta|t|/2})$ and $(e^{-3\beta|t|/2}, e^{\beta|t|/2})$ are ambiguous and the first is homogeneous in time.

∎

Remarks:

The decay $e^{-\beta|t|}$ is of Debye type which is observed in polar liquids (including H_2O and biofluids). It is only products of first time derivatives of μ_{2i} and $\vec{J}_{fi}$ which must have decay faster than $o(1/t)$ at large times for scattering theory to be well-defined. The corresponding magnetic field $\vec{H}$ behaves consistently since $\vec{\nabla} \times \vec{J}_c$ is perpendicular to $\hat{n}_J$ and $\vec{\nabla} \cdot \vec{H}$ is set by continuity of the tangential fields. This gives a specific example of the ambiguity mentioned in ref. (13).

We have been unable to construct similar examples for perfect dielectrics and in simple cases such as Debye's exponential time dependence times a polynomial or a Guassian time decay times another polynomial for ϵ_2 and $\vec{E}$ gives uniqueness when the uniform Hölder continuity at the boundary is taken into account. Since this occurs in such limited class of dielectrics and fields it will not be presented here.

The examples homogenization from spatially dependent constitutive functions are especially anisotropic because of terms such as $(\vec{\nabla}\mu_2)$ and $(\vec{\nabla}\epsilon_2)$. Limited angle scattering is even more questionable than usual for these profiles. Pure metals and perfect dielectrics with only spatial dependence with continuous boundaries satisfy Müller's uniqueness theorm[11] and this can be relaxed to Lipschitz continuous at the boundary with exponent α, $0 \leq \alpha \leq 1$, using results from Colton and Kress[12]. Hence, any fixed angle of incidence or limited angle ambiguity is weak and easily removed.

Now, the scaling structure will be given by substituting eqs. (5a - d) and eqs. (6a, b) into Maxwell's equations. Again the curl of the curl equations will be rearranged using the assumption that (μ_2, ϵ_2) depend only on time or space separately. For time dependence only, the electric field satisfies the wave equation

$$\begin{aligned}\Delta\vec{E}^{\sim} - \mu_1\epsilon_1\ddot{\vec{E}}^{\sim} = {} & \vec{\nabla}(\vec{\nabla}\cdot\vec{E}^{\sim}) + \mu_s\partial_t(\frac{\mu_2}{\mu_s}\vec{J}_f^{\sim}) + (\mu_2\epsilon_2 - \mu_1\epsilon_1)\ddot{\vec{E}}^{\sim} + [\dot{\mu}_2\epsilon_2 + 2\mu_2\dot{\epsilon}_2 - \mu_2\epsilon_2\frac{\dot{\mu}_s}{\mu_s}]\dot{\vec{E}}^{\sim} \\ & + \left[(\dot{\mu}_2\dot{\epsilon}_2 + \mu_2\ddot{\epsilon}_2) - (\dot{\mu}_2\epsilon_2 + 2\mu_2\dot{\epsilon}_2)\frac{\dot{\mu}_s}{\mu_s} - \mu_2\epsilon_2\left\{\frac{\ddot{\mu}_s}{\mu_s} - 2(\frac{\dot{\mu}_s}{\mu_s})^2\right\}\right]\vec{E}^{\sim} \qquad (11a)\end{aligned}$$

and the corresponding magnetic field satisfies the wave equation

$$\begin{aligned}\Delta\vec{H}^{\sim} - \mu_1\epsilon_1\ddot{\vec{H}}^{\sim} = {} & \vec{\nabla}(\vec{\nabla}\cdot\vec{H}^{\sim}) - \vec{\nabla}\times\vec{J}_f^{\sim} + (\mu_2\epsilon_2 - \mu_1\epsilon_1)\ddot{\vec{H}}^{\sim} + \left[\mu_2\dot{\epsilon}_2 + 2\dot{\mu}_2\epsilon_2 - 2\mu_2\epsilon_2\frac{\dot{\mu}_s}{\mu_s}\right]\dot{\vec{H}}^{\sim} \\ & + \left[(\dot{\mu}_2\dot{\epsilon}_2 + \ddot{\mu}_2\epsilon_2) - (\mu_2\dot{\epsilon}_2 + 2\dot{\mu}_2\epsilon_2)\frac{\dot{\mu}_s}{\mu_s} - \mu_2\epsilon_2\left\{\frac{\ddot{\mu}_s}{\mu_s} - 2(\frac{\dot{\mu}_s}{\mu_s})^2\right\}\right]\vec{H}^{\sim}. \qquad (11b)\end{aligned}$$

The structure of Maxwell's equations gives a somewhat simpler, more symmetric, pair of equations for the case of a spatial dependent scaling function, $\mu_s(\vec{x})$. The substitution of the scaling equations in eqs. (5), (6) into Maxwell's equations gives the pde for the electric field as

$$\Delta\vec{E}^{\sim}-\mu_1\epsilon_1\ddot{\vec{E}}^{\sim} = \vec{\nabla}(\vec{\nabla}\cdot\vec{E}^{\sim})+\mu_2\dot{\vec{J}}_f^{\sim}+(\mu_2\epsilon_2-\mu_1\epsilon_1)\ddot{\vec{E}}^{\sim}-\nabla\times[\frac{1}{\mu_s}(\vec{\nabla}\mu_s)\times\vec{E}^{\sim}]+[\vec{\nabla}\mu_2+\frac{\mu_2}{\mu_s}(\vec{\nabla}\mu_s)]\times\dot{\vec{H}}^{\sim}. \quad (12a)$$

The pde which the magnetic field satisfies is

$$\Delta\vec{H}^{\sim}-\mu_1\epsilon_1\ddot{\vec{H}}^{\sim} = \vec{\nabla}(\vec{\nabla}\cdot\vec{H}^{\sim})-\vec{\nabla}\times\vec{J}_f^{\sim}+(\mu_2\epsilon_2-\mu_1\epsilon_1)\ddot{\vec{H}}^{\sim}-\vec{\nabla}\times[\frac{1}{\mu_s}(\vec{\nabla}\mu_s)\times\vec{H}^{\sim}]-[\vec{\nabla}\epsilon_2+\frac{\epsilon_2}{\mu_s}(\vec{\nabla}\mu_s)]\times\dot{\vec{E}}^{\sim}. \quad (12b)$$

The coupling between the electric and magnetic fields in eqs. (12a, b) is identical to those in eqs. (8a, b).

The time-dependent scaling case is richer than homogenization case.

Example 2 To eliminate the dissipation in the electric field $\vec{E}$ in the general case of time dependent scaling with (ϵ_2, μ_2) time dependent only, eq. (11a) requires that

$$\dot{\mu}_2\epsilon_2 + 2\mu_2\dot{\epsilon}_2 - 2\mu_2\epsilon_2\frac{\dot{\mu}_s}{\mu_s} = 0.$$

Since $\mu_2\epsilon_2 \neq 0$, it can be divided through the previous equation to give

$$\frac{\dot{\mu}_2}{\mu_2} + 2\frac{\dot{\epsilon}_2}{\epsilon_2} - 2\frac{\dot{\mu}_s}{\mu_s} = 0.$$

This can be integrated directly to yield,

$$\mu_s(t) = c_3\sqrt{\mu_2(t)}\epsilon_2(t),$$

which is the most general scaling allowed for (ϵ_2, μ_2) dependent on time only.

■

Similarly eq. (11b) is dissipation-free for $\vec{H}$ if

$$\mu_2\dot{\epsilon}_2 + 2\dot{\mu}_2\epsilon_2 - 2\mu_2\epsilon_2\frac{\dot{\mu}_s}{\mu_s} = 0$$

which can be solved the same way to give the most general scaling for dissipation-free $\vec{H}$ as

$$\mu_s(t) = c_4\sqrt{\epsilon_2(t)}\mu_2(t).$$

■

It is also possible to have a weak ambiguity where additional measurements can restore uniqueness.

The space - dependent scaling case is complicated and coupled.

Example 3 The condition of dissipation-free fields when $(\mu_s, \mu_2, \epsilon_2)$ depend on space only can be solved in general. From eq. (12b)

$$\frac{1}{\epsilon_2}(\vec{\nabla}\epsilon_2) + \frac{1}{\mu_s}(\vec{\nabla}\mu_s) = \vec{0}$$

if $\epsilon_2\epsilon_s \neq 0$ then this equation can be integrated directly in the direction of the gradients to obtain

$$\epsilon_2(\vec{x})\mu_s(\vec{x}) = c_1$$

for each $c_1 \in R^1$. Hence, there are ambiguities for dissipation-free propagation.

■

The existence of trapped modes for an obstacle, at least in two space dimensions, can be described by repeatedly performing Darboux transformations on a scattering operator

for the object without any interior eigenvalue. Each transformation produces a factor of modulus one in frequency domain,

$$\frac{k+i\lambda_j}{k-i\lambda_j}=e^{i\alpha_j}, \tag{13a}$$

so that N applications gives

$$\prod_{j=1}^{N}\left(\frac{k+i\lambda_j}{k-i\lambda_j}\right)=e^{i(\alpha_1\ +\cdots+\ \alpha_N)}. \tag{13b}$$

In terms of the initial reflection operator $R_0(k)$, the same obstacle with N-trapped interior modes becomes

$$R_N(k)=e^{i(\alpha_1\ +\cdots+\ \alpha_N)}R_0(k) \tag{13c}$$

where λ_j is the wave number of the j^{th} trapped mode. The Fourier transform of eq. (13c) with $ck=\omega$ gives the time-fomain reflection operator. Thus an ambiguity occurs if only the modulus squared of the reflection is measured. It is a weak ambiguity for a classical wave because if the wave amplitude is measured the phase can be determined.

4. CONCLUSIONS

The non-uniqueness of the time-domain electromagnetic scattering problem was discussed. Two kinds of non-uniqueness for inverse problems, a stong ambiguity which cannot be removed by additional measurements and a weak ambiguity which can be removed by appropriate additional data were discussed and a few examples were given.

In electromagnetic inverse scattering theory a recent paper by on of us, BDF[9], was simplified and discussed. Working with one scaling function led to nicer equations but two were required to show that only one of them is independent. Two general features involving the Poynting vector of scaled interior solution $\vec{S}^{\sim}$ and another showing the invariance of the conductivity under the scaling transformations. This actually explains an observation of Kabanikhin and Lorenzi[13]. The difference in ambiguities due to different non-constant coefficient functions, called homogenization, and ambiguities due to scaling, called the Sabatier scaling or electromagnetic impedance, were derived and examples were presented. Finally, these examples have the potential to act as checks on the correctness of computer programs for electromagnetic scattering .

It would be interesting to use new technological tools such as molecular beam epitaxy (MBE) to use these ideas to attempt to fabricate new non-dissipative materials. Biological entities may already be using the idea in Examples 2 and 3.

Electromagnetic scattering with variable permitivity, permeability and conductivity is a rich enough model that the surface is barely scratched.

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